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Effect of turbulent Prandtl number on the computation of film-cooling effectiveness

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ABSTRACT

This paper presents a method to improve the accuracy of computed film cooling effectiveness spanwise distribution. The effect of turbulent Prandtl number in the flow field outside the near-wall region on the computation is studied. Realizable k–ε model with a one-equation model in near-wall region is employed. The results show that the variation of turbulent Prandtl number has great influence on the computation. Reducing turbulent Prandtl number increases film cooling effectiveness of the whole spanwise region remarkably under large blowing ratios. Under small blowing ratios, the reduction of turbulent Prandtl number decreases the cooling effectiveness of the center region, and increases the effectiveness of the lateral region off the centerline. Compared with the single value turbulent Prandtl number computation the agreement between computation and measured results is improved notably with varied one in different spanwise regions. A new laterally varying turbulent Prandtl number (LV-Pr_t) model dependent on lateral location and blowing ratio has been suggested. Computation accuracy is improved greatly by LV-Pr_t model. Compared with the TLVA-*Pr* model of Lakehal [D. Lakehal, Near-wall modeling of turbulent convective heat transport in film cooling of turbine blades with the aid of direct numerical simulation data, ASME J. Turbomach. 124 (2002) 485–498] which provides the best results in the calculated cases LV-Pr_t model is an effective way to improve computation accuracy in the frame of the traditional isotropic turbulence models. More work on the information of the turbulent Prandtl number has to be done for the further development of the LV-Pr_t model.

1. Introduction

Land-based industrial gas turbines are commonly operated continuously over long operational hours. This places severe demands on component life for such engines, especially the components working in high temperature environment, such as turbine rotors and vanes. However, increasing requirement on high efficiency requires higher turbine rotor inlet gas temperature. Consequently, cooling of gas turbine components is inevitable, and film cooling is widely used as an effective means to maintain component temperatures at acceptable levels.

Nowadays numerical simulation on film cooling investigation is getting more and more important, because of its low-cost, duration-reduced, labor-saved and complete-data characteristics. Along with the rapid development of computer and CFD technology, the accuracy of film cooling simulation from three-dimension N–S equations has improved greatly. But a problem still exists that the accuracy of the computed cooling effectiveness spanwise distribution or the jet spreading is not good enough [1,3,13]. The computed effectiveness is frequently overpredicted in the near-centerline region and is underpredicted in the bilateral region. The main reason responsible for this inaccuracy is that the turbulence models and turbulent parameters used in the computation can not describe the turbulence characteristics of film cooling precisely. Turbulence models employed in film cooling calculations in general do not go beyond the isotropic two-equation scope; see Lakehal et al. [12] for a review. Recently Hoda and Acharya [13] conducted a study where various closures for turbulent stresses were applied for the prediction of coolant jet in crossflow. The models employed ranged from high and low-*Re* number k–ε and k–ω models to nonlinear eddy viscosity variants. Their contribution led to the conclusion that this type of closure needs further improvements because turbulence in film-cooling flows – more generally the jets in crossflow – is considerably anisotropic. Kaszeta et al. [2] measured the mean velocity and turbulent shear stress for the mixing region of a film cooling situation in which the coolant was streamwise injected with an injection angle of 35°. Values for the turbulent viscosities μ_t in the spanwise direction and wall-normal direction were calculated. Results showed that the turbulent viscosity μ_t in the spanwise direction is larger than that in the wall-normal direction. It is known that the isotropic turbulence models can not account for this anisotropic characteristic and underpredict the jet lateral spreading consequently. Azzi et al. [3] developed a direct numerical simulation (DNS) based two-layer approach that combined an anisotropic one-equation model in

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Nomenclature

D	film cooling hole diameter
M	blowing ratio (=$\rho_c U_c/\rho_g U_g$)
L	film cooling hole length
Re_y	near-wall turbulent Reynolds number $\left(=\rho y_n\sqrt{k}/\mu\right)$
x	streamwise coordinate originating at the trailing edge of cooling hole
z	spanwise coordinate originating at the trailing edge of cooling hole
y	normal coordinate
y_n	normal wall distance
y^+	dimensionless wall distance $\left(=\rho y_n\sqrt{\tau_w/\rho}/\mu\right)$
Pr_t	turbulent Prandtl number (=$\mu_t/\rho\Gamma_t$)
Pr	molecular Prandtl number (=$\mu/\rho\Gamma$)
k	turbulent kinetic energy
$\overline{u_i}$	averaged velocity components
Δ	ratio of ($\eta_{0.4}-\eta_{0.85}$) to $\eta_{0.85}$
T	temperature

Greek symbols

η	film cooling effectiveness (=$(T_{aw}-T_g)/(T_c-T_g)$)
ρ	density
μ	molecular viscosity
μ_t	turbulent viscosity
Γ	molecular heat diffusivity
Γ_t	turbulent heat diffusivity
$\ell_\mu/\ell_\varepsilon$	length scales in one-equation model
τ_w	wall friction
$\eta_{0.4}$	local film cooling effectiveness computed with Pr_t = 0.4
$\eta_{0.85}$	local film cooling effectiveness computed with Pr_t = 0.85

Subscripts

aw	adiabatic wall
c	jet
g	mainstream

near-wall region with an isotropic k–ε model. Comparison with the measurement results showed that the computation accuracy was improved greatly by this new version approach.

As we know, the turbulent heat diffusivity Γ_t is expressed as $\mu_t/\rho Pr_t$. Although the isotropic turbulence models can not calculate different values of μ_t in different directions is an important reason for the underprediction of lateral effectiveness, but it may not be the only reason in author's opinion. Turbulent Prandtl number Pr_t can be another important factor. For convection heat transfer computation Pr_t is very important, because its value affects the accuracy of heat diffusion directly. Many researchers had paid attention to the variation of Pr_t in the boundary layer and its effect on the heat transfer calculation. Kays [4] made a detailed review and analysis on this issue. For film cooling computation, Lakehal [5] included the variation of Pr_t in the computation and made the computed effectiveness spanwise distribution more close to the measurement. However, his work only considered the Pr_t variation in the inner core of boundary layer ($y^+ \leqslant 94$). In the outer region where $y^+ > 94$, $Pr_t = 0.76$ is used.

According to Kays's study [4], Pr_t tends to decrease to values of 0.5–0.7 in the boundary layer's outer layer, and the behavior of Pr_t in this region was of less importance in calculating heat transfer. So, Pr_t in the whole outer flow field could take the same value with the "log" region which is 0.85 for gas. This value is often employed in the film cooling computation. It should be noticed that film cooling flow field is different from the normal boundary layer. It includes not only the boundary layer on the wall, but also the mixing layer (or the shear layer) and the jet (see Fig. 1). One major difference between these regions and the inner core of boundary layer is the value of Pr_t. Mayer and Divoky's [15] tabulation of Pr_t values in axisymmetric and plane turbulent jets and wakes indicated a range of 0.42–0.83. Wygnanski and Fiedler [16] investigated the heat transport in a slightly heated two-dimensional shear layer and suggested a value of around 0.5 for Pr_t. Fiedler [17] reported a smaller value of Pr_t in the almost fully turbulent region of the two-dimensional shear layer later. Townsend [18] proposed a variation of Pr_t with the total strain, and the minimum value of Pr_t indicated by his scheme was 0.4. Chambers and Antonia [19] measured the turbulence field in a thermal mixing layer associated with a turbulent plane jet and a value of 0.4 was calculated. For axisymmetric round jets, $Pr_t = 0.7$ was suggested by Chidambaram et al. [20] and Henze [21]. For film cooling flow field, only Kohli and Bogard [14] reported some information of

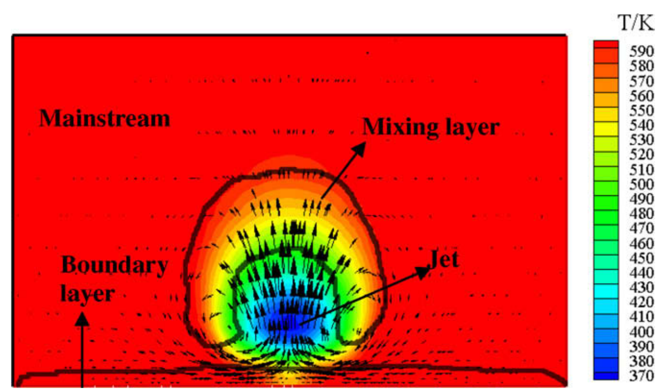

Fig. 1. Temperature contours and velocity vectors in the plane vertical to the flow.

Pr_t in the mixing layer region and the jet region. Although the information was not very plenty, it still showed that the value of Pr_t was different from the usually used value around 0.9.

To make the computation agree well with the experiment, in addition to the way that has been tried by Azzi et al. [3] to develop anisotropic turbulence models, another way is to assign proper value or variation of Pr_t for film cooling flow field as recommended by Lakehal [5]. However, Lakehal [5] only consider the Pr_t distribution in the inner core of boundary layer and the results show that anisotropic turbulence model still plays a major role in the improvement. This paper extends the idea of Lakehal [5] and pays attention to the value of Pr_t in mixing layer and jet region. It is known that the cooling effectiveness spanwise distribution is influenced strongly by the temperature field in mixing layer and jet region, which is dependent on the ability of the jet lateral spread. So the turbulent parameters, such as Pr_t, in these regions are very important to computation accuracy. The aim of this paper is to investigate how the variation of Pr_t in mixing layer and jet region influences the computed cooling effectiveness spanwise distribution and whether it can improve the accuracy.

2. Computational approach

2.1. Computational models and boundary conditions

Goldstein et al. [6] measured the film cooling effectiveness downstream of a long cylindrical hole inclined at 35° in the streamwise direction. In his work, the flow field quality of the wind tunnel was carefully verified. And all the measurement instruments were also calibrated with good precision. The mainstream boundary layer velocity profile and its displacement thickness, the velocity profile in the film hole were measured. The data in reference [6] are of great reliability and are employed as the benchmark to varify the calculation of this paper.

The geometry of the computation zone, which has the same size with the tested model in reference [6], is illustrated in Fig. 2a. The width of computational zone is half of the model, because it is symmetrical relative to the spanwise middle plane, and symmetry boundary condition is imposed at there. The cooling hole diameter D is 2.35 cm and the length-to-diameter ratio L/D is 30, which would be able to guarantee the flow in the hole fully developed and its velocity profile very close to the measurement. The length of the channel upstream to the hole exit center is 22 D. The computed boundary layer displacement thickness at the streamwise position of the cooling hole exit center is equal to the measurement in the condition of without cooling jet.

The experiments were carried out with four blowing ratios ranging from $M = 0.1$ to 2 in the investigation of [6]. In the present work the computations are carried out for three typical blowing ratios: $M = 0.5$, $M = 1$ and $M = 1.5$. Following flow parameters are set to correspond with the experiment: mainstream velocity at entrance of the channel $U_g = 61$ m/s for $M = 0.5$ and 1, $U_g = 30.5$ m/s for $M = 1.5$, mainstream temperature $T_g = 308$ K, jet temperature $T_c = 363$ K. Perfect gas equation is applied in the calculation of the flow densities with the pressure of 101,325 Pa and their temperatures. The mass-flux inlet boundary condition is set at the entry of the cooling hole and the value of mass flux is computed based on the values of U_g, M, ρ_g and ρ_c. Inlet turbulent intensities of the mainstream and the jet are 0.5% and 2%, respectively. The

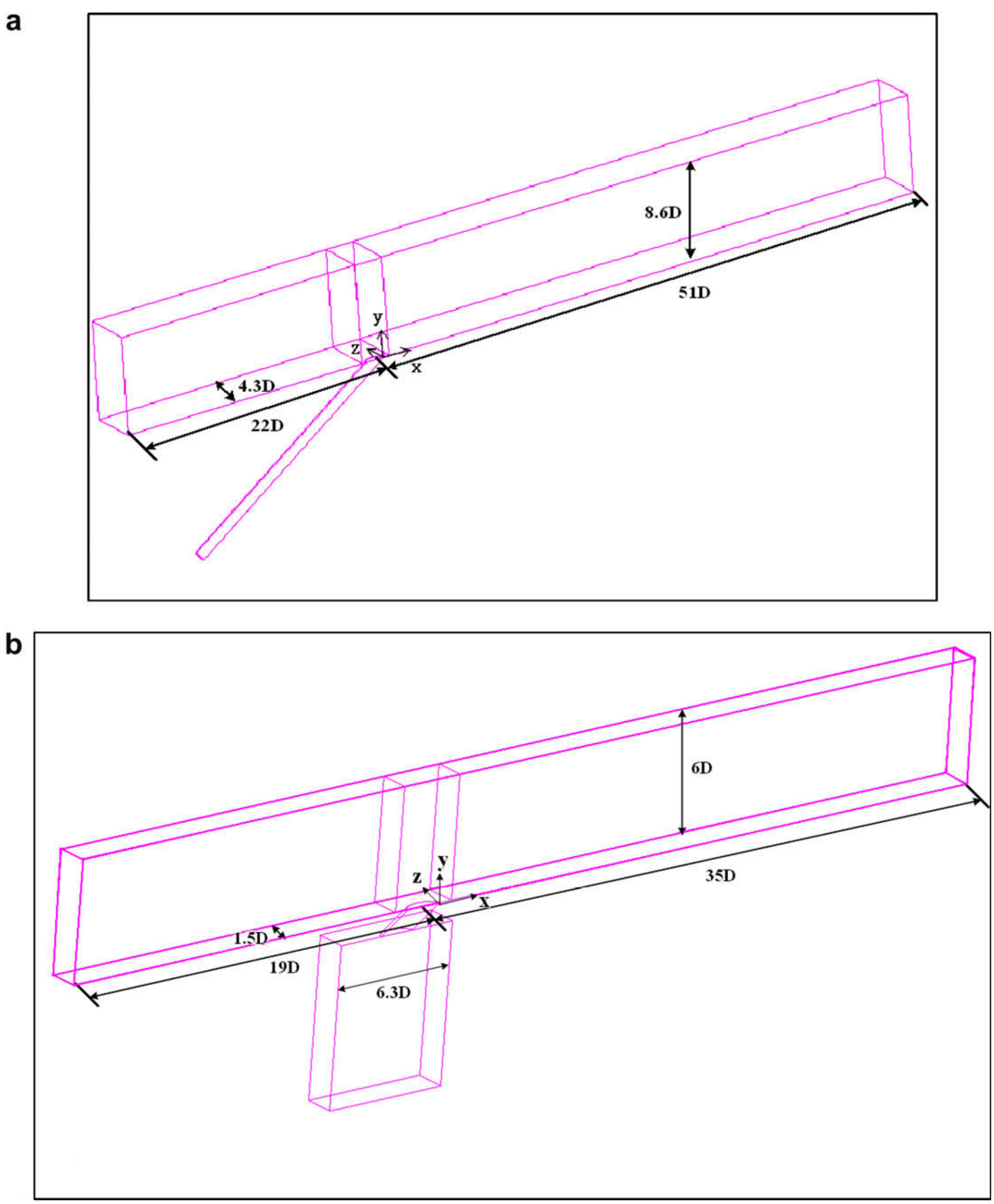

Fig. 2. Geometry of the computational model (a) Simulating Goldstein et al. [6] (b) Simulating Sinha et al. [23].

thermal boundary condition on the tested wall is adiabatic and its temperature is therefore the adiabatic wall temperature T_{aw}.

However, the experimental model used in Goldstein et al. [6] just had a single hole and the hole length was much longer than that in the modern gas turbine airfoil. So a more realistic model with a shorter hole length, a plenum, and a row of holes, should be calculated with the method of this paper. Another model computed in this paper is that studied by Sinha et al. [23]. It was a single row of film cooling hole on a flat plate. The holes were inclined at a angle of 35° with a lateral spacing of 3 D. The hole diameter was 12.7 mm and the length-to-diameter ratio L/D was 1.75. The Computational model which is designed according to [23] is illustrated in Fig. 2b. Symmetry conditions are imposed on the plane through the middle of the hole ($z/D = 0$) and the plane at $z/D = 1.5$ in the middle of two neighboring holes. Sinha et al. [23] studied two density ratios ($\rho_c/\rho_g = 1.2$ and 2) and various blowing ratios from 0.25 to 1. In this paper the calculation is carried out in the condition of $\rho_c/\rho_g = 2$ and $M = 0.5$ to compare with the approach of [5]. Compressibility effect is taken into account through use of equation of state. The mainstream inlet conditions are set as: $U_g = 20$ m/s, $T_g = 302$ K, and $T_u = 0.5\%$. Mass-flux inlet boundary condition is set at the inlet surface of the jet plenum with $T_c = 153$ K and $T_u = 2\%$. Adiabatic wall conditions are also set on the walls. The above boundary conditions coincide with those in [23] and [5].

2.2. Governing equations and turbulence model

The conservative form of the three-dimensional steady state compressible flow Reynolds-averaged Navier–Stokes equations can be written as

$$\frac{\partial}{\partial x_k}(\rho \overline{u_k}) = 0 \tag{1}$$

$$\frac{\partial}{\partial x_j}(\rho \overline{u_i}\overline{u_j}) = -\frac{\partial \bar{p}}{\partial x_i} + \frac{\partial}{\partial x_j}\left[\mu \frac{\partial \overline{u_i}}{\partial x_j} + \mu_t\left(\frac{\partial \overline{u_i}}{\partial x_j} + \frac{\partial \overline{u_j}}{\partial x_i}\right) - \frac{2}{3}\delta_{ij}\rho k\right] \tag{2}$$

$$\frac{\partial}{\partial x_j}(\rho \overline{u_j}\overline{T}) = \frac{\partial}{\partial x_j}\left[\left(\frac{\mu}{Pr} + \frac{\mu_t}{Pr_t}\right)\frac{\partial \overline{T}}{\partial x_j}\right] \tag{3}$$

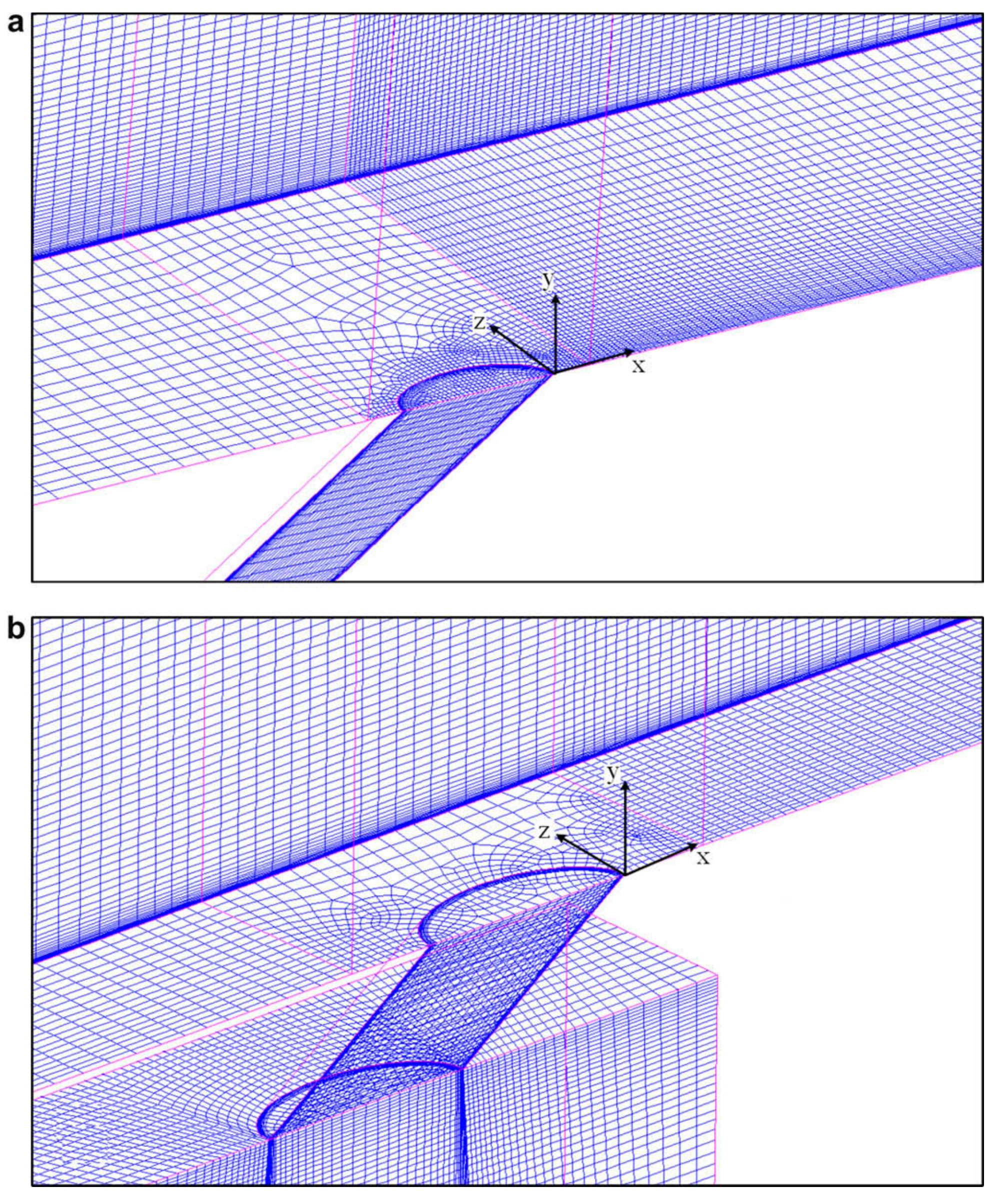

Fig. 3. Mesh details in the hole and the region near the wall.

where $\bar{p}$ represents the pressure, $\overline{T}$ the averaged temperature, δ_{ij} the Kronecker delta. μ_t is the turbulent viscosity computed with turbulence model equations.

In order to exhibit the effect of Pr_t variation outside the viscosity-affected near-wall region ($Re_y < 200$) on the computed film cooling effectiveness spanwise distribution, it is necessary to exclude the improvement brought by using the anisotropic turbulence model and the Pr_t variation inside the viscosity-affected near-wall region. In this paper an isotropic realizable k–ε model of Shih et al. [7] is used to compute the flow field outside the viscosity-affected near-wall region ($Re_y < 200$). This model satisfies the so-called realizability constraints for the Reynolds stresses, specifically requiring positivity of the Reynolds stresses and satisfaction of Schwarz's inequality for the shear stresses. A constant value of Pr_t is used inside the viscosity-affected near-wall region. Many researchers [8,9] employed this model and showed that it performed better with two-layer approach than the other turbulence models such as the standard k–ε model and SST k–ω model.

The two-layer approach is essentially that the whole flow domain is divided into a viscosity-affected region ($Re_y < 200$, including laminar sublayer, buffer region and logarithmic region) and a fully-turbulent region, and in the fully-turbulent region the realizable k–ε model is employed and in the viscosity-affected region an isotropic one-equation model which can resolve the viscosity-affected region including laminar sublayer (typically $y^+ \approx 1$) is employed. In the one-equation model, the Eddy viscosity is made proportional to a velocity scale and a length scale ℓ_μ. The distribution of ℓ_μ is prescribed algebraically while the velocity scale is determined by solving the k-equation. The dissipation rate ε appearing as sink term in the k-equation is related to k and a dissipation length scale ℓ_ε which is also prescribed algebraically. The different two-layer versions available in the literatures differ in the use of the velocity scale and the way to prescribe the length scales ℓ_μ and ℓ_ε as shown in [12,22]. The one equation model [10,11] employed here reads

$$\mu_t = C_\mu \rho \sqrt{k} \ell_\mu; \quad \ell_\mu = y_n C_\ell^* (1 - e^{-Re_y/A_\mu}) \tag{4}$$

$$\varepsilon = k^{3/2}/\ell_\varepsilon; \qquad \ell_\varepsilon = y_n C_\ell^* (1 - e^{-Re_y/A_\varepsilon}) \tag{5}$$

$$C_\ell^* = \kappa C_\mu^{-3/4}, \ \ A_\mu = 70, \ \ A_\varepsilon = 2C_\ell^*, \ \ C_\mu = 0.09 \tag{6}$$

where κ is Von Karman constant. The turbulent Prandtl number is set separately in the two regions. In the viscosity-affected region

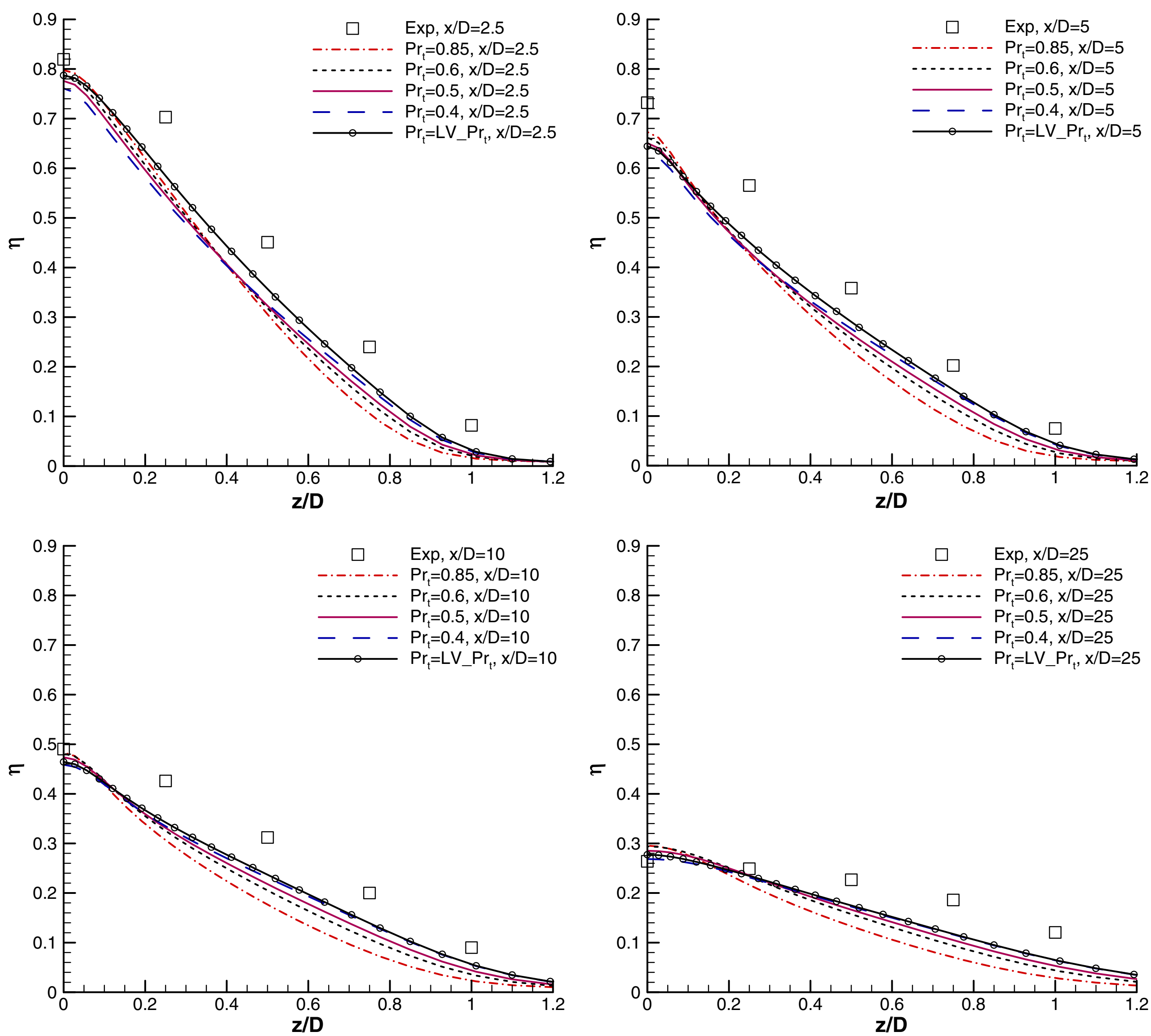

Fig. 4. Spanwise distributions of local film cooling effectiveness for $M = 0.5$: comparison between experiment results of Goldstein et al. [6] and calculations.

($Re_y < 200$), Pr_t is set as 0.85 which is a typical and approved value in the boundary layer heat transfer calculation for gas [4]. According to references [14–21], the value of Pr_t in the outer flow field is in the range of 0.4–0.83. In our computation, four values are investigated: 0.4, 0.5, 0.6 and 0.85. The value of 0.85 is that widely used in film cooling calculations.

2.3. Grids

A multi-block grid is used in this study to allow the highest quality in all regions with the fewest number of cells. In this way, the computational domain can be partitioned into several subsections: cooling hole, the plenum and the other three in the channel (Fig. 2). Each section is meshed with an appropriate topology. In order to resolve the mean velocity, mean temperature, heat flux and turbulent quantities in the viscosity-affected near-wall region ($Re_y < 200$) accurately, the near-wall turbulence model requests that the y^+ value at the wall-adjacent cell should be the order of one and there are at least ten cells in this region. So in the hole and the region near the test wall the density of cells is densified to satisfy this requirement. After a series of tests and adjustments the final adopted grid for calculations is obtained. Grid dependence studies show that grid-independence results can be obtained with $210 \times 85 \times 44$ nodes in x, y and z directions of the channel and $40 \times 80 \times 56$ nodes in the hole for the computational of the model of Goldstein et al. [6]. The corresponding node numbers for the calculation of the model of Sinha et al. [23] are $200 \times 80 \times 26$, $30 \times 35 \times 42$, $80 \times 60 \times 15$ for channel, hole and plenum, respectively. Finer meshes only result in negligible changes, which are less than 1%, in the calculation of centerline film cooling effectiveness downstream the film cooling hole for both computational models. Fig. 3 shows some mesh details in the hole and the near-wall region for the two computational models.

2.4. Computational method

The governing equations are solved by a three-dimensional finite-volume method for arbitrary nonorthogonal grids. The discretization scheme for all the transport equations' convection terms is QUICK scheme which is of third order precision. The diffusive fluxes are approximated using the central-difference scheme.

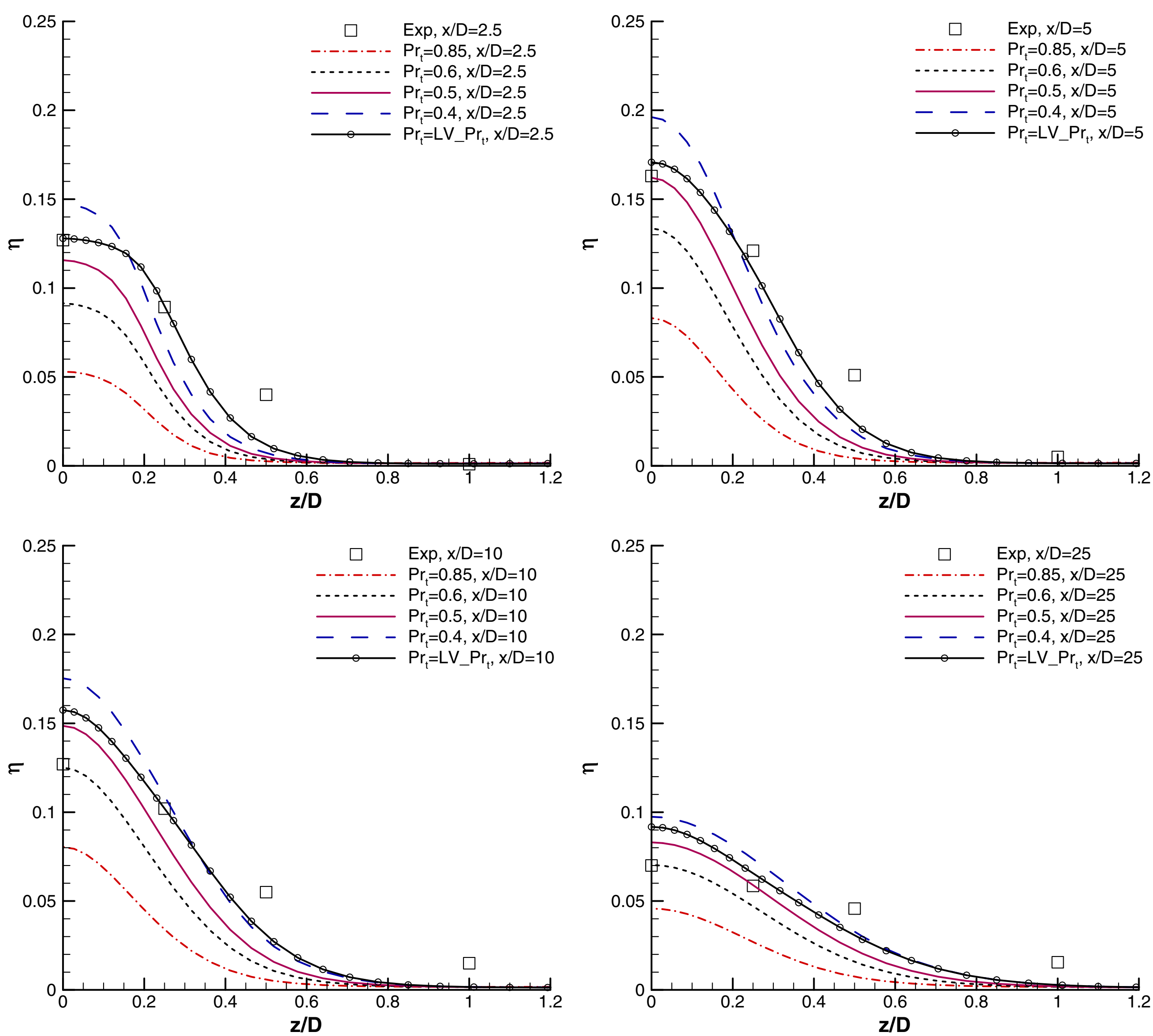

Fig. 5. Spanwise distributions of local film cooling effectiveness for $M = 1.5$: comparison between experiment results of Goldstein et al. [6] and calculations.

The velocity-pressure coupling is achieved by using the well-known SIMPLEC algorithm. All the difference equations are solved sequentially with the implicit procedure algorithm. And the convergence in all cases is determined based on a drop in normalized mass and momentum residuals by four orders of magnitude (10^{-4}) at least and the mass-weighted average temperature in the y–z plane at the streamwise position $X/D = 10$ changes less than 0.1% with increasing iterations. The algebraic multi-grid method is employed to speed up the converging process.

3. Comparisons with the data of Goldstein et al. [6] and discussion

Figs. 4–6 represent the measured values of Goldstein et al. [6] and the computed cooling effectivenesses distribution in the spanwise direction with different turbulent Prandtl numbers at four streamwise positions for $M = 0.5$, $M = 1$ and $M = 1.5$, respectively. Fig. 7 shows the comparison of the calculated temperature contours in the y–z plane at the streamwise position $X/D = 10$ with different values of Pr_t.

3.1. M = 0.5

Fig. 4 shows lateral variation of the cooling effectiveness for both experiment and calculation. It can be seen that in all the streamwise positions decreasing Pr_t increases the cooling effectiveness in the lateral region $0.3 < z/D < 1.2$, but it reduces in the region near the centerline to some extent. From the definition of turbulent heat diffusivity: $\Gamma_t = \mu_t/\rho Pr_t$, it is clear that reduction of Pr_t could enhance the heat diffusion ability, which means that the lateral heat flux from the jet center to its border is increased and consequently the temperature in the jet center region is decreased and increased at the border. The calculated temperature contours in y-z plane are shown in Fig. 7 for $Pr_t = 0.85$ and 0.5. It can be seen in Fig. 7a that part of the wall is covered by the jet center flow. Jet center temperature of large Pr_t is obviously higher than that of small Pr_t in the near wall region, however, the behavior of the comparison of the flow temperature for different Pr_t becomes conversely at the places off the centerline. This is the mechanism of the cooling effectiveness of large Pr_t becoming higher than that of small Pr_t in the center region and lower at the lateral places off the centerline.

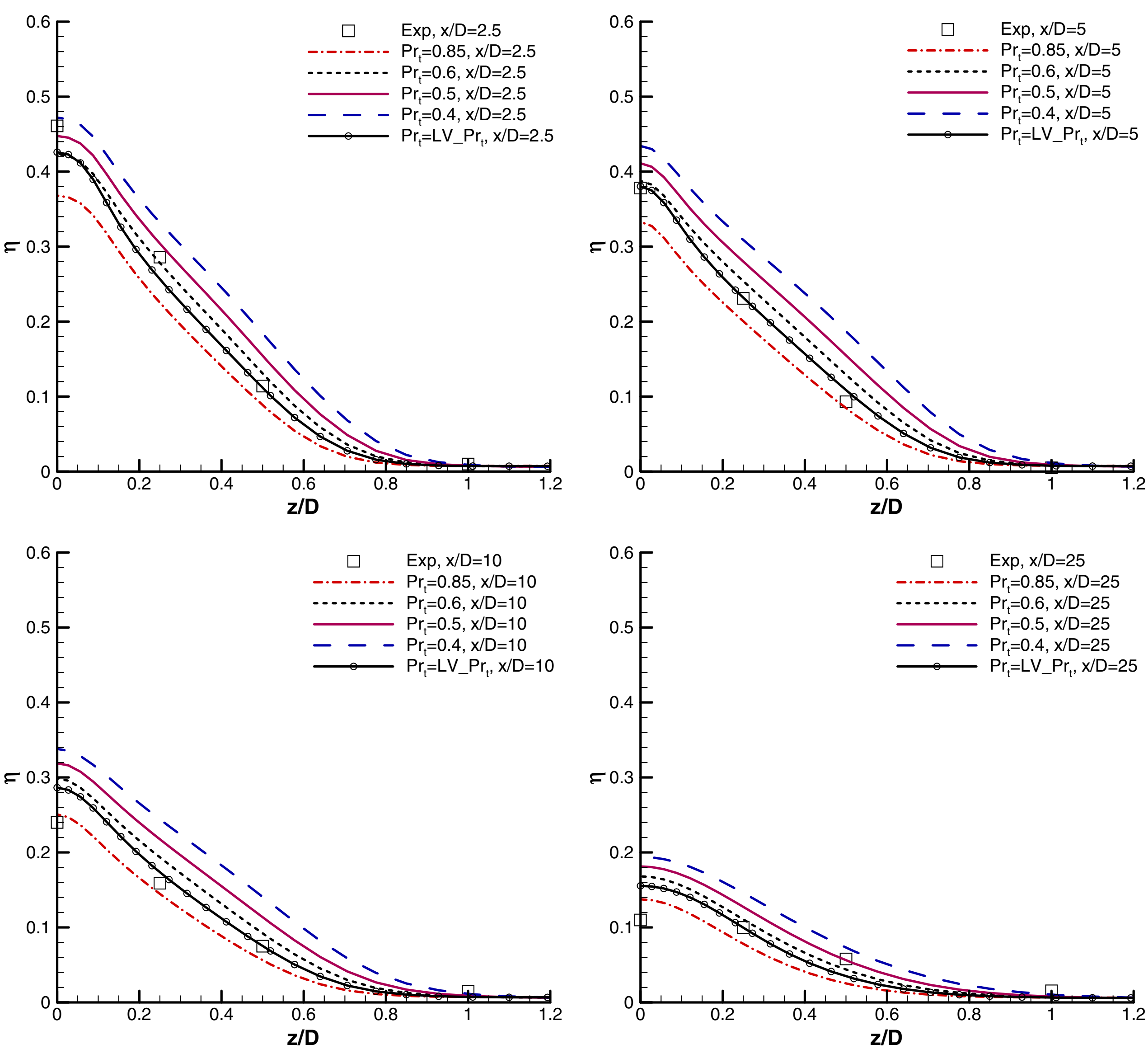

Fig. 6. Spanwise distributions of local film cooling effectiveness for $M = 1$: comparison between experiment results of Goldstein et al. [6] and calculations.

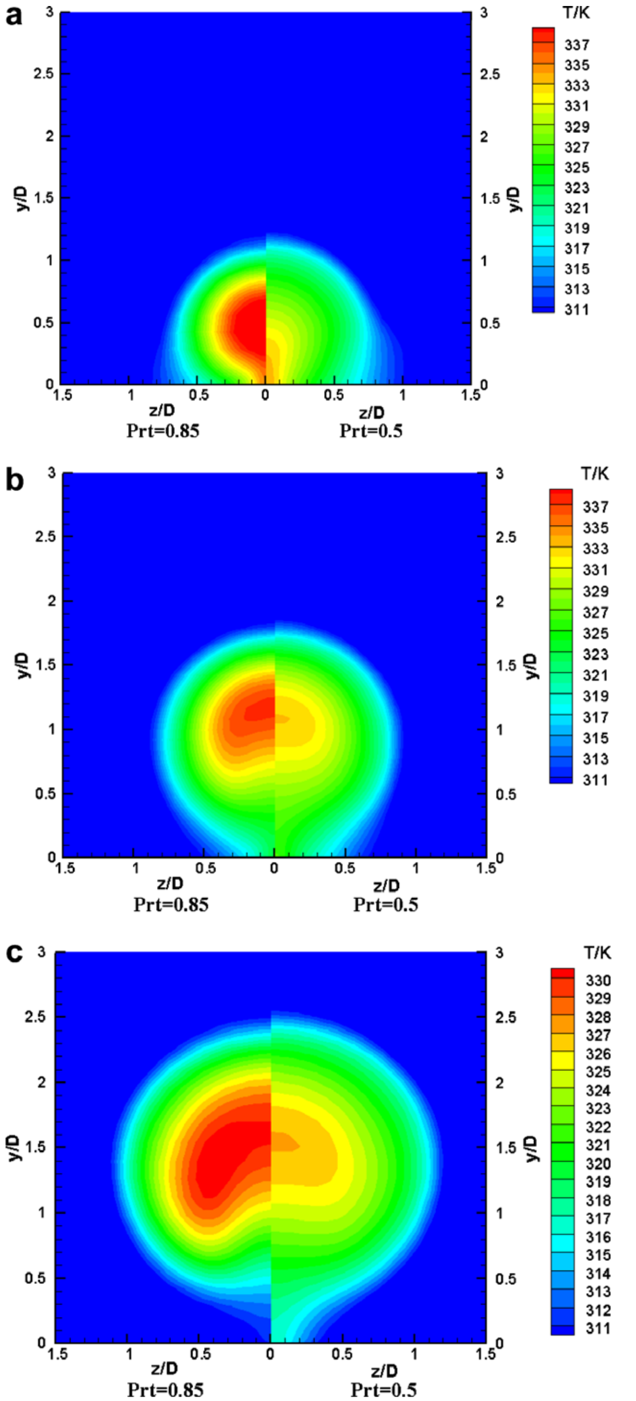

Fig. 7. Temperature contours in the y–z plane at $X/D = 10$: calculations with two turbulent Prandtl numbers (a) $M = 0.5$ (b) $M = 1$ (c) $M = 1.5$.

Fig. 4 also shows that the agreement between calculation and experiment is getting better with smaller turbulent Prandtl number in the lateral region $0.3 < z/D < 1.2$ at all the streamwise positions. However, better results are obtained in the near-centerline region with large turbulent Prandtl numbers. The reason for this regional feature can be found in Fig. 7a. The wall is not only covered by the mixing layer region, but also by the jet center region where Pr_t is relatively larger. Although decreasing Pr_t enhances the heat diffusion in the spanwise direction, the heat diffusion in the streamwise direction is also increased by the employed isotropic turbulence model and is beyond the real extent. This makes the temperature in the near-centerline region lower than the real. If we want to improve the computation accuracy in the whole spanwise region, accurate anisotropic turbulence models that can calculate the real values of turbulent viscosity μ_t in different directions have to be developed and accurate Pr_t in the film cooling flow field should be measured.

3.2. M = 1.5

It is different from the case of $M = 0.5$ that decreasing Pr_t makes the cooling effectiveness increase in the whole $0 < z/D < 1$ region notably at all the streamwise positions as shown in Fig. 5. To compare the values of calculated maximum effectiveness for $Pr_t = 0.85$ and 0.4 it can be found that the increment of the effectiveness due to decreasing Pr_t takes almost the same value of around 0.1 for the case of $M = 1.5$ and $M = 0.5$. However, the relative cooling effectiveness increment defined as the ratio of $(\eta_{0.4} - \eta_{0.85})$–$\eta_{0.85}$: $\Delta = (\eta_{0.4} - \eta_{0.85})/\eta_{0.85}$ is different. As shown in Fig. 8 the increment of relative effectiveness for $M = 1.5$ is much larger than that of $M = 0.5$ at most of the lateral location. This indicates that the variation of Pr_t has greater influence on the effectiveness in large blowing ratio conditions. Fig. 8 also shows that the maximum increment of effectiveness for $Pr_t = 0.85$ happens a the lateral location near half width of the jet and it is located close to the jet border for $Pr_t = 0.4$. The main reason for this is that the jet separates from the wall almost completely in large blowing ratio and can only affect the wall in a small spanwise region through heat diffusion (see Fig. 7c). Heat diffusion acts as a major role in large blowing ratio film cooling. The reduction of Pr_t enhances heat diffusion. So the spanwise region influenced by jet is expanded, and the wall temperature and the effectiveness are increased notably from large Pr_t to small Pr_t in the same spanwise position relatively. Although decreasing Pr_t also enhanced heat diffusion in spanwise direction for small blowing ratio condition, the relative enhancing effect is not notable because the jet of small blowing ratio affects the wall in a large spanwise region through covering mainly (see Fig. 7a) and heat diffusion is only a minor part.

Fig. 5 also shows that all the computed effectiveness of $Pr_t = 0.85$ and 0.6 are obviously lower than the measured. Better agreement could be achieved with $Pr_t = 0.4$ and $Pr_t = 0.5$. Nevertheless the agreement is not perfect that the calculated values are lar-

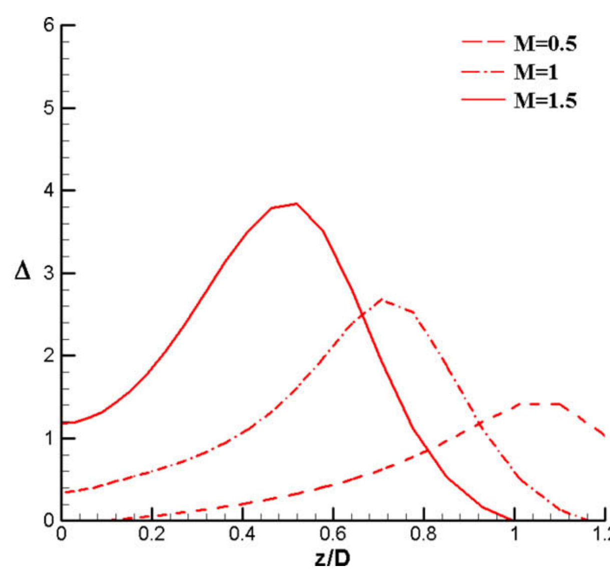

Fig. 8. The curves of Δ along the spanwise direction at the streamwise position $X/D = 10$.

ger than the measured at centerline and lower than it at outer lateral locations. This situation implies that the isotropic turbulence models used in this paper is not able to simulate the turbulence characteristics of film cooling properly if a single value of turbulent Prandtl number is applied throughout the calculated domain. As reported in [2] that the turbulent transport in the spanwise direction is stronger than in the wall-normal direction [2] and the isotropic model underpredicts the spanwise heat diffusion.

The clue got from the above discussion is that a varied turbulent Prandtl number with respect to space would be able to give better agreement between the computation and measurement. In Fig. 5, it can be seen that in the near-centerline region $z/D < 0.1$ the results of $Pr_t = 0.6$ and $Pr_t = 0.5$ are in the best agreement with the measured values; in the lateral region $0.1 < z/D < 0.3$ the results of $Pr_t = 0.5$ are in the best agreement with the measured values; in the lateral region $z/D > 0.3$ the results of $Pr_t = 0.4$ are in the best agreement with the measured values. This is a reflection of that in the flow field of film cooling the value of Pr_t is not a single constant, but a variable of space position. If we want to get good computation results by employing isotropic turbulence models, the accurate information of Pr_t in the flow field is necessary. And we will give more discussion on this issue in the following sections.

Table 1
Advised spanwise turbulent Prandtl number distribution (LV-Pr_t) for different blowing ratios

$M = 0.5$	z/D	0	0.1	$\geqslant 0.3$
	Pr_t	0.85	0.8	0.4
$M = 1$	z/D	0	0.1	$\geqslant 0.3$
	Pr_t	0.4	0.55	0.7
$M = 1.5$	z/D	0	0.2	$\geqslant 0.3$
	Pr_t	0.8	0.75	0.4

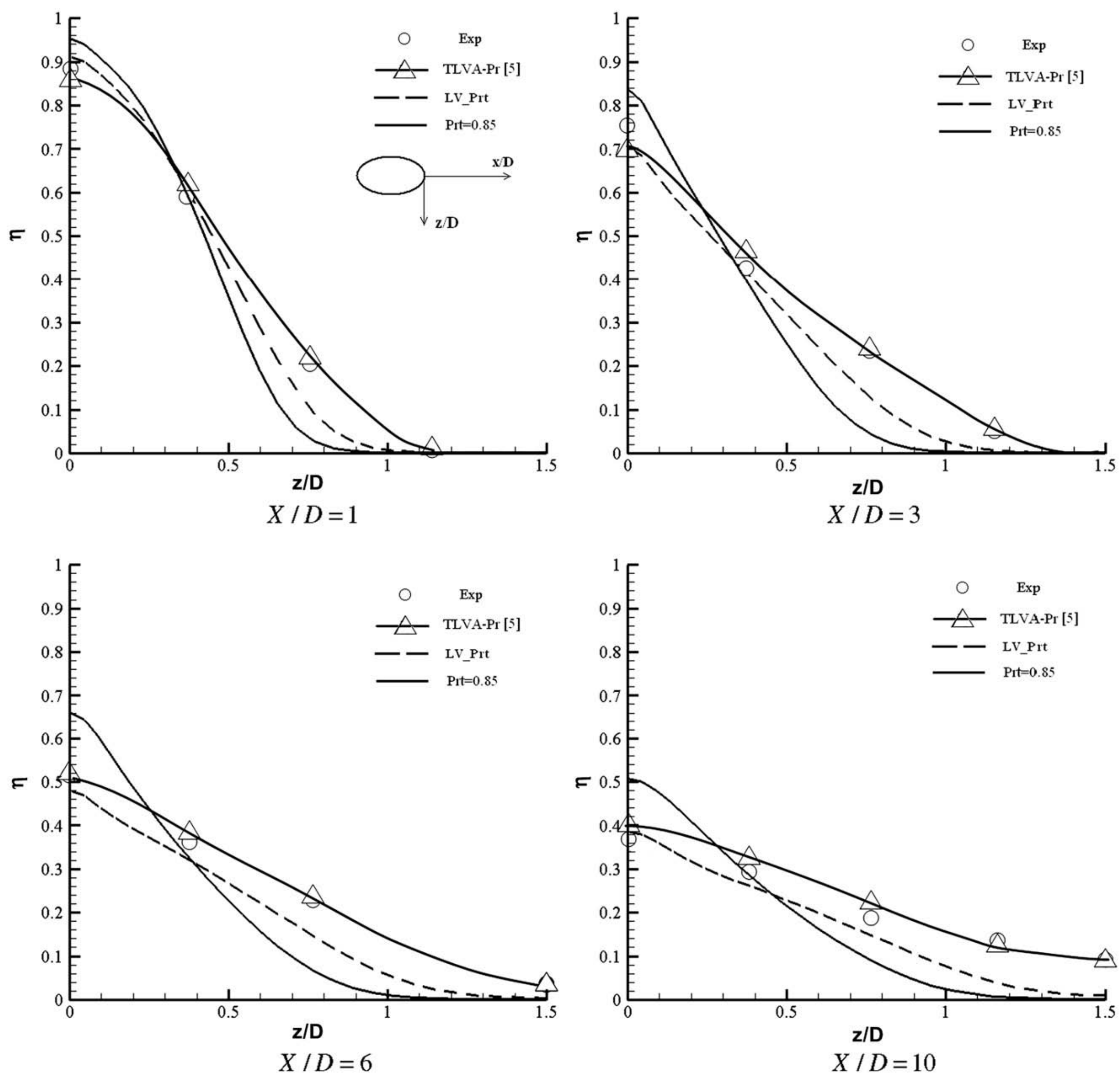

Fig. 9. Spanwise distributions of local film cooling effectiveness for $M = 0.5$: comparison between experiment results of Sinha et al. [23] and calculations.

3.3. $M = 1$

Through the above discussion and comparing the three temperature contours in Fig. 7, we can know that the wall influenced by the jet of $M = 0.5$ is in the jet center region and mixing layer region; the wall influenced by the jet of $M = 1.5$ is in the outer boundary of mixing layer region. However, in case of $M = 1$, the influenced wall is just in the mixing layer region. The enhanced heat diffusion from jet center to the lateral by decreasing Pr_t increases the cooling effectiveness in the whole spanwise region for $M = 1$ case (see Fig. 6). This is similar with $M = 1.5$ case. But due to the influenced wall is covered by the mixing layer region in case of $M = 1$, heat diffusion doesn't play a major role as in case $M = 1.5$. And the curve of $\Delta = (\eta_{0.4} - \eta_{0.85})/\eta_{0.85}$, which represents the influence degree of changing Pr_t on computed results, in Fig. 8 for $M = 1$ is between the other two curves.

From Fig. 6 we can find that better correspondence between computed results and experiment results is obtained by small Pr_t in the upstream near-centerline region, and large Pr_t does better work in the lateral off the centerline. But in the downstream, large Pr_t does better work in the near-centerline region and small Pr_t gives good computation in the lateral. This regional feature is different from the other two cases. In authors' opinion, this is caused by the spreading of jet in the y–z plane which is vertical to the flow direction. In the upstream region only the wall near the centerline is covered by the mixing layer, and the other part of wall is out of mixing layer. So decreasing Pr_t improves the computation in the near-centerline region, but overpredicts in the lateral (see Fig. 6). As flowing downstream, the jet spreads in the y–z plane and more lateral wall is covered by the mixing layer. However, the jet center where Pr_t is relatively large is near to the wall gradually. And this change makes the regional feature of Pr_t in the downstream region different from the upstream region. Again we come to the viewpoint obtained from Section 3.2 that Pr_t is a variable of space position in the film cooling flow field.

3.4. Laterally varying turbulent Prandtl number (LV-Pr_t)

Although the above comparisons with the experiment data show that the variation of Pr_t in both stream and spanwises would be needed to improve the agreement. But it is found that the effect of the variation of Pr_t in spanwise is more important. Based on the careful analysis of the data a lateral varying turbulent Prandtl number model (LV-Pr_t) is suggested, which is listed in Table 1. The specific values of turbulent Prandtl number is dependent on two parameters: lateral location in the jet and blowing ratio. Linear interpolation or polynomial fit can be used to calculate the Pr_t value at the positions out of the table. The calculated results with this model and comparisons with the data of [6] are shown in Figs. 4–6 for $M = 0.5$, 1.0 and 1.5, respectively. The agreement with the data has improved significantly for all the test conditions. Some slight discrepancy still appears at the location off the centerline particularly in the condition of $M = 0.5$.

4. Comparisons with the data of Sinha et al. [23] and discussion

Lakehal [5] employed TLVA-*Pr* method, which is the combination of anisotropic two-layer turbulence model and the DNS-based model of Pr_t in the boundary layer, in calculation and comparison with the experiment of Sinha et al. [23] ($\rho_c/\rho_g = 2$ and $M = 0.5$). Very good agreement with the data has achieved as shown in Fig. 9. In this figure the calculated effectiveness with single value of turbulent Prandtl number $Pr_t = 0.85$ and LV-Pr_t model listed in Table 1 are also plotted. It is obvious that the model of $Pr_t = 0.85$ significantly overestimate the effectiveness at centerline and underestimate at the location relative far from it. The model of LV-Pr_t gives much better results than the model of $Pr_t = 0.85$ does. The spanwise cooling effectiveness distribution is calculated well using LV-Pr_t, especially in the region $z/D < 0.5$. The agreement in the lateral outer region of the wall jet is not good enough as expected. It is clear that the TLVA-*Pr* method of [5] still creates the best results in the whole spanwise region, especially in the region far from the centerline $z/D > 0.5$. LV-Pr_t model is an effective way to improve the computation accuracy of film cooling in the frame of the commonly used isotropic turbulence models. More work related to the information of turbulent Prandtl number in the outer region of the wall jet would be needed for the further development of the model LV-Pr_t.

5. Conclusion

The aim of this work is to investigate the effect of varying turbulent Prandtl number in the outer region of the boundary layer on the computation of film cooling effectiveness spanwise distribution. Besides developing anisotropic turbulence model it could be a new way to improve the computation accuracy of film cooling. The results show that changing Pr_t has great influence on the computation of effectiveness. Great improvement can be achieved with laterally varying Pr_t even in the frame of the traditional isotropic turbulence models.

Decreasing turbulent Prandtl number enhances heat diffusion capacity in the flow. The cooling effectiveness of large blowing ratio is increased notably in almost all the spanwise region because heat diffusion plays a major role in this case. For small blowing ratio, decreasing Pr_t reduces the effectiveness in the near-centerline region and increases the effectiveness in the lateral region of the centerline. The effect of varying Pr_t is larger in the case of high blowing ratio. In order to get good agreement with the experiment a regional values of Pr_t is necessary. It is found the lateral variation of Pr_t is more important than that of stream wise. A spanwise location and blowing ratio dependent model of Pr_t is given. The accuracy of effectiveness prediction has improved with this model especially in the region close to the centerline. Some discrepancy still exists in the area relative far from the centerline. TLVA-*Pr* model of Lakehal [5] still provides the best results in the calculated cases. More attention about the information of Pr_t in the outer region of wall jet should be paid in the further development of this model.

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